Entanglement and flow
and what they say about
Universality and Simulateability of QC

Damian Markham

Joint work with Elham Kashefi

CNRS, LTCI–ENST (Telecom ParisTech), Paris
Motivation

• Where do the quantum advantages lie in quantum computation?
  (what’s so quantum about it?)

• Is this connected to quantum effects in nature?
  (quantum computation as a phase transition?)
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• Where do the quantum advantages lie in quantum computation? (what’s so quantum about it?)

• Is this connected to quantum effects in nature? (quantum computation as a phase transition?)

Unify results and ideas

Universality & Classical Simulatability  VS  Entanglement & Flow (structural prop of MBQC)

Unify results and ideas
• **Universality**

  *Can it be used to do universal QC?*
  
  - Circuit made up of Rotation and CNOT
  - MBCQ: resource state $|\Phi_n\rangle$ satisfies
    
    $$E(|\Phi_n\rangle) > \log n$$
    
    [Van den Nest et al. 06/07]

• **Classical simulatability**

  *Can it be simulated efficiently on a classical computer?*

  - Clifford gates only [Gottesman, Knill 97]
  - Match gates with n.n. [Valiant 02, Van den Nest 08, Jozsa Miyake 08]
  - Bounded number of 2 qubit gates [Jozsa 06]
  - MBQC: efficiently sim. if
    
    $$E(|\Phi_n\rangle) < \log n$$

  [Shi. et al, Yoran Short, Van den Nest et al. 05]
Results

• Relationship

Entanglement ↔ Flow

• New conditions for classical simulatability
  (in terms of flow and entanglement)

• Recover several known results
  - Clifford gates only [Gottesman, Knill 97]
  - Bounded number of 2 qubit gates [Jozsa 06]
Measurement Based Quantum Computation (MBQC)

- Initial entangled resource state $|\Phi_n\rangle$
- Single qubit measurements
- Local corrections

R. Raussendorf and H. J. Briegel, PRL 85 5188 (2001)
D. Browne and H. J. Briegel, quant-ph/0603226
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Measurement is random
(need to recover determinism)
Measurement Based Quantum Computation (MBQC)

- What are the correction operations, and *why do we need them*?

Measurement is random  
(need to recover determinism)

Measure first qubit

\[ P \quad 1-P \]
Measurement Based Quantum Computation (MBQC)

- What are the correction operations, and **why do we need them?**

Measurement is random
(need to recover determinism)

Measure first qubit
Measurement Based Quantum Computation (MBQC)

• What are the correction operations, and why do we need them?

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Measure first qubit
Measurement Based Quantum Computation (MBQC)

- **What are the correction operations**, and why do we need them?

  Flow tells us when it can be done, and which operations to make.
Flow / gFlow

• Condition for a graph state to allow deterministic computation
  (tells you if corrections exist and what they are)

Mathematically, there is gFlow if there exists both
- Function \( f : O^c \rightarrow P^l_c \) (arrow)
- Partial order \( \leq \) (measurement sequence)

1) \( i \notin f(i) \) and \( i \in odd(f(i)) \)
2) if \( j \in f(i) \) and \( i \neq j \) then \( i < j \)
3) if \( j \leq i \) and \( i \neq j \) then \( j \notin odd(f(i)) \)

Flow / gFlow

• Condition for a graph state to allow deterministic computation
  (tells you if corrections exist and what they are)

• Strong structural approach gives rise to

  - Depth complexity gap between MBQC and Circuit model
    [Broadbent, Kashefi 07, Browne, Kashefi, Perdrix 09]

  - New translations between circuit and MBQC
    [de Beautrap et al 08]

Entanglement interpretation of flow

- Need $n$ ebits across any cut In : Out
  (since LOCC computation can be seen as teleportation)
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- Need n-ebits across any cut In : Out → n wires
Entanglement interpretation of flow

- Need n-ebits across any cut In : Out → n wires
- Extra edges? -> shouldn’t break n-ebits

Flow / g-flow

n wires

+ no ‘bad’ cycles
Graphical conditions: gFlow

- Function \( f : O^c \rightarrow P^l_c \) (arrow)
- Partial order \( \leq \) (measurement sequence)

1) \( i \not\in f(i) \) and \( i \in \text{odd}(f(i)) \)
2) if \( j \in f(i) \) and \( i \neq j \) then \( i < j \)
3) if \( j \leq i \) and \( i \neq j \) then \( j \not\in \text{odd}(f(i)) \)

1 ebit only!

An open graph \((G, I, O)\) permits a unitary if it has gflow
Universality / Simulatability in MBQC

• What is a universal resource state / when does a resource state give classical simulatable MBQC?

  • Universality

    \[ E(\Phi_n) > \log n \]

    [Van den Nest et al. 06/07]

  • Classical simulatability

    - Any MBQC can be simulated in

    \[ O(n^2 \text{poly}(2^{E(\Phi_n)}) \) \]

    - Efficiently simulatable if

    \[ E(\Phi_n) < \log n \]

    [Shi. et al, Yoran Short, Van den Nest et al. 05]
Structural Entanglement

\[ E_{struc}(|\Phi\rangle) := \min_{Order} \max_{Cut,k} E_{AB}(|\Phi\rangle) \]

[Yoran & Short 05]
Structural Entanglement

\[ E_{\text{struc}}(\Phi) = \min_{\text{Order} 1,...,n} \max_{k} E_{AB}(\Phi) \]

[Yoran & Short 05]
Structural Entanglement

\[ E_{\text{struc}}(\Phi) := \min_{\text{Order}} \max_{\text{Cut} k, A=1..k, B=k+1..n} E_{AB}(\Phi) \]

[Yoran & Short 05]
Structural Entanglement

\[ E_{struc}(\Phi) := \min_{\text{Order }} \max_{\text{Cut } k} E_{AB}(\Phi) \]

[Yoran & Short 05]

\[ K=17 \]

\[ E_{AB}(\Phi) \]
Structural Entanglement

\[ E_{struc}(\Phi) = \min_{\text{Order}} \max_{\text{Cut}} E_{AB}(\Phi) \]

[Yoran & Short 05]
Universality / Simulatability in MBQC

• What is a universal resource state / when does a resource state give classical simulatable MBQC?

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[Shi. et al, Yoran Short, Van den Nest et al. 05]
Flow and structural entanglement

- Use flow to bound the entanglement
  - Flow gives a ‘natural’ order

\[ E_{\text{struc}} (\Phi) = \min_{\text{Order}} \max_{\text{Cut'k}} E_{AB} (\Phi) \]

\[ E_{AB} (\Phi) = \frac{1}{n} \sum_{i=1}^{n} A_i B_i \]
Flow and structural entanglement

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- gFlow implies wires in -> out
Flow and structural entanglement

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• Use to give an order
  left to right, top to bottom
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• gFlow implies wires in -> out
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  entanglement ~ number of wires cut
Flow and structural entanglement

• Use flow to bound the entanglement
  - Flow gives a ‘natural’ order

\[ E_{\text{struc}}(\Phi) = \min_{\text{Order}} \max_{\text{Cut }k} E_{AB}(\Phi) \]

\[ E_{AB}(\Phi) = 3 \]

• \text{gFlow} implies wires in -> out

• Use to give an order
  left to right, top to bottom

• Choose cut maximising entanglement
  entanglement ~ number of wires cut

• Since it may not be the \textit{minimum} order:

\[ E_{\text{struc}}(\Phi) \leq 1 + \max \text{ no. edges between Flow wires} \]
Flow and structural entanglement

• Use flow to bound the entanglement
  - Flow gives a ‘natural’ order
  - Flow implies wires in -> out

\[ E_{struc}(|\Phi\rangle) := \min_{Order} \max_{Cut \ k \ A=1..k \ B=k+1..n} E_{AB}(|\Phi\rangle) \]

• gFlow implies wires in -> out
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• Since it may not be the minimum order:

\[ E_{struc}(|\Phi\rangle) \leq 1 + \max \text{no. edges between Flow wires} \]
Flow and Classical Simulatability in MBQC

• Given a gFlow, call

\[ C_F = \max \text{ no. edges crossing between gFlow wires } \]

- Any MBQC can be simulated in

\[ O(n^2 \text{poly}(2^{C_F})) \]

- Efficiently simulatable if

\[ C_F < \log n \]
Connection to Jozsa condition

• If the number of 2qubit gates in a circuit is $D$
  [Jozsa 06]

  - It can be simulated in
    $$O(n^2 \text{poly}(2^D))$$
  - Efficiently simulatable if
    $$D < \log n$$
Connection to Jozsa condition

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  [Jozsa 06]
  - It can be simulated in
    \[ O(n^2 \text{poly}(2^D)) \]
  - Efficiently simulatatable if
    \[ D < \log n \]

• Follows from map circuit -> MBQC

\[ \text{Input} \quad \rightarrow \quad \text{Output} \]
Conclusions

• New condition for simulatability in terms of flow / entanglement
  - Any MBQC can be simulated in $O(n^2 \text{poly}(2^{C_F}))$
  - Efficiently simulatable if $C_F < \log n$

• Unified (somewhat) set of conditions
  (flow, entanglement, universality, simulatability)

• Can it all be described as entanglement?
• Connection to criticality?
Thank you!

http://iq.enst.fr/