Measurement Based Quantum Computing on Fractal Lattices

Damian Markham
CNRS
Telecom ParisTech (E.N.S.T.)

Janet Anders, Michal Hajdusek, Vlatko Vedral

How much of one way computation is *just* thermodynamics?

Damian Markham
CNRS
Telecom ParisTech (E.N.S.T.)

Janet Anders, Michal Hajdusek, Vlatko Vedral

Found. Phys. 38, 506 (2008)

How much of one way computation is *just* thermodynamics?

Damian Markham
CNRS
Telecom ParisTech (E.N.S.T.)

Janet Anders, Michal Hajdusek, Vlatko Vedral

Superlattices and Microstructures, Vol. 23, No. 3/4, 1998

How much of physics is *just* computation?

TOMMASO TOFFOLI†
ECE Department, Boston University, 44 Cummington St, Boston, MA 02215, U.S.A.

We explore an emergent interpretation of the action integral and the lagrangian in physics, and discuss its connection with the concept of 'amount of computation'. We give an abstract definition of action, and argue (1) that it provides a general, model-independent characterization of 'amount of computation'; and that (2) the action of physics is a special case of this general action. Much as entropy quantifies the lack of information one has about the *state* of a system, action quantifies the lack of information about the system's *law*—or, equivalently, its *behavior*. In this approach, action is to dynamics what entropy is to statics.

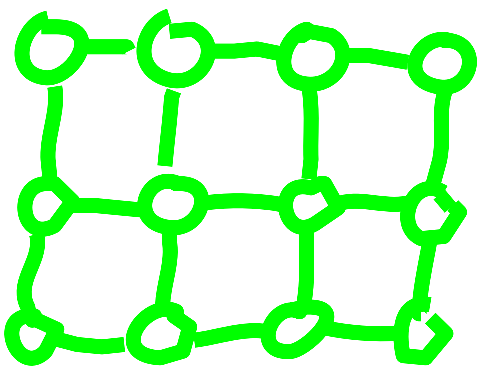
Spin

MBQC

No
Phase Transition

Not
Universal

Phase Transition

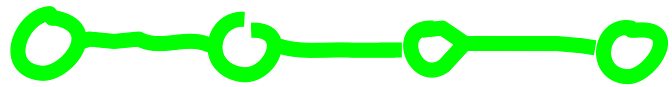


Universal

Spin

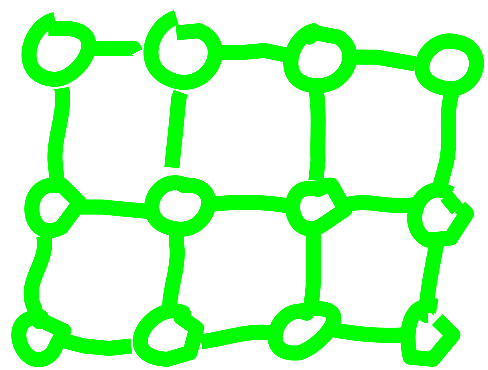
MBQC

No
Phase Transition



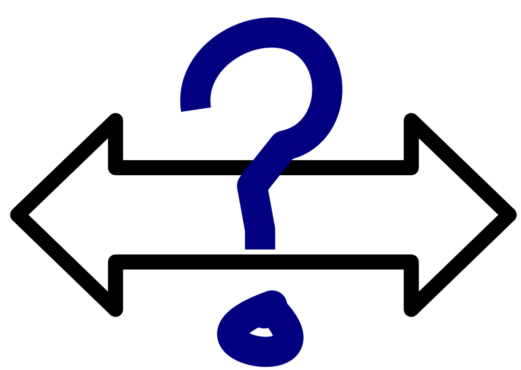
Not
Universal

Phase Transition



Universal

phase transition in spin



universality for MBQC

Overview

- The analogy
- Examples (fractal lattices)
- Proofs for fractal lattices

What is a Phase Transition?

Small changes in parameter, global property change

H_2O

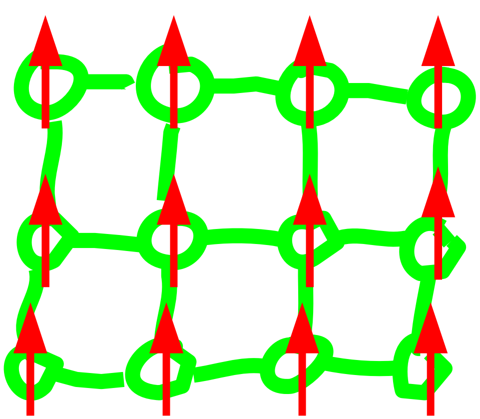
Ice
-0.1°C

T_C=0°C

Water
0.1°C

Spin

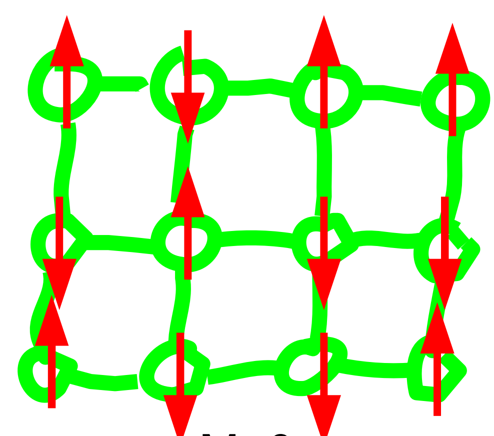
Ordered



M>0

T_C

Disordered



M=0

No Phase Transition in 1D
(Peierls)

- Nature minimizes Free Energy (2^{nd} Law)

$$F = U - T S$$

No Phase Transition in 1D

(Peierls)

- Nature minimizes Free Energy (2nd Law)

Temp

$$F = U - TS$$

Energy

Entropy = log (number of ways of using E)

'Spread' energy as much as possible
(for each temp T)

No Phase Transition in 1D
(Peierls)

- Nature minimizes Free Energy (2nd Law)

Temp

$$F = U - TS$$

Energy

Entropy = log (number of ways of using E)

'Spread' energy as much as possible
(for each temp T)

Can a state be 'ordered' at finite temperature? ⟺ Does it have minimum Free Energy?

No Phase Transition in 1D
(Peierls)

- Nature minimizes Free Energy (2nd Law)

Temp

$$F = U - TS$$

Energy

Entropy = log (number of ways of using E)

'Spread' energy as much as possible
(for each temp T)

Can a state be 'ordered' at finite temperature? ⇔ Does it have minimum Free Energy?

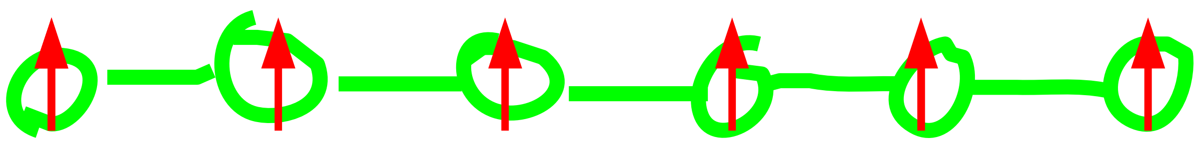
Shake it and see!

No Phase Transition in 1D
(Peierls)

- Take ordered state, and check if min F, for some T>0

shake ⟺ FLIP some spins (break order)

1D

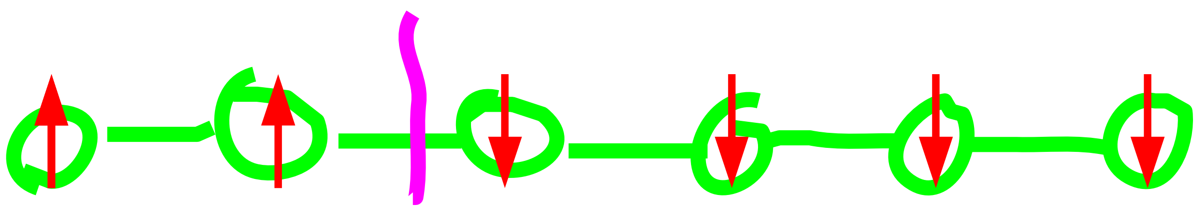


No Phase Transition in 1D
(Peierls)

- Take ordered state, and check if min F, for some T>0

shake ⟺ FLIP some spins (break order)

1D



$\Delta U = \langle H_{\text{ordered}} \rangle - \langle H_{\text{flip}} \rangle = 2J$

$\Delta S = \log(\text{no. ways to spend } \Delta U) = \log(N)$

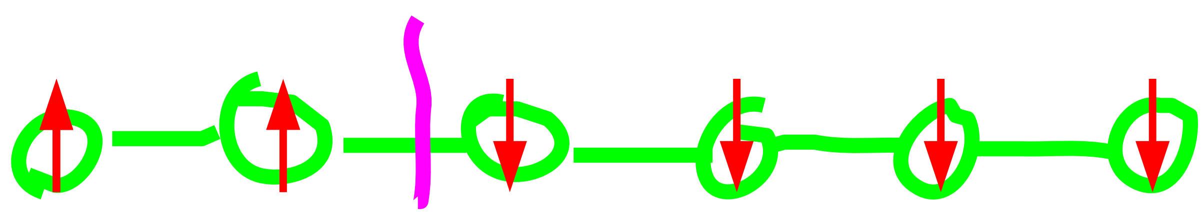
$$\Delta F = 2J - T \log(N)$$

No Phase Transition in 1D
(Peierls)

- Take ordered state, and check if min F, for some T>0

shake ⟺ FLIP some spins (break order)

1D



$\Delta U = \langle H_{ordered} \rangle - \langle H_{flip} \rangle = 2J$

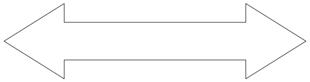
$\Delta S = \log(\text{no. ways to spend } \Delta U) = \log(N)$

$\Delta F = 2J - T \log(N)$

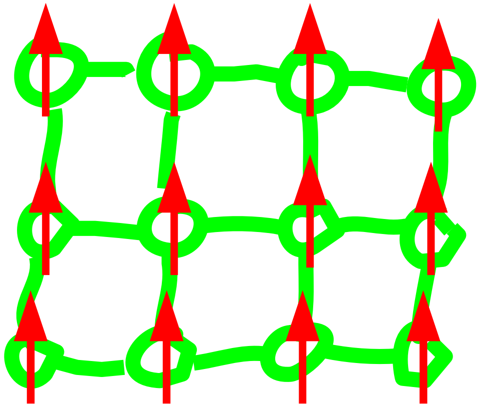
T -> 0 as
N -> ∞

No Phase Transition in 1D
(Peierls)

- Take ordered state, and check if min F, for some T>0

shake  FLIP some spins (break order)

2D

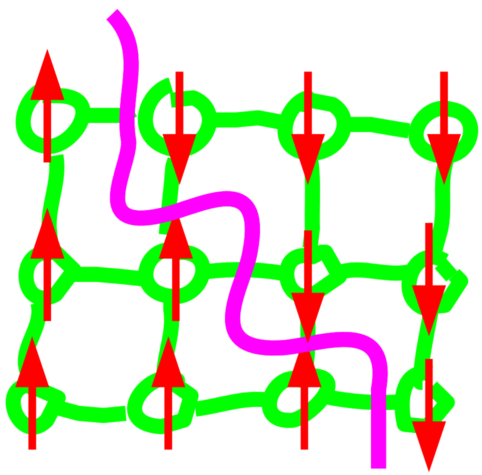


No Phase Transition in 1D
(Peierls)

- Take ordered state, and check if min F, for some T>0

shake ⟺ FLIP some spins (break order)

2D



$\Delta U = \langle H_{ordered} \rangle - \langle H_{flip} \rangle = 2NJ$

$\Delta S = \log(\text{no. ways to spend}) = \log(3\text{^}N)$

$\Delta F = 2NJ - T\,N\log(3)$

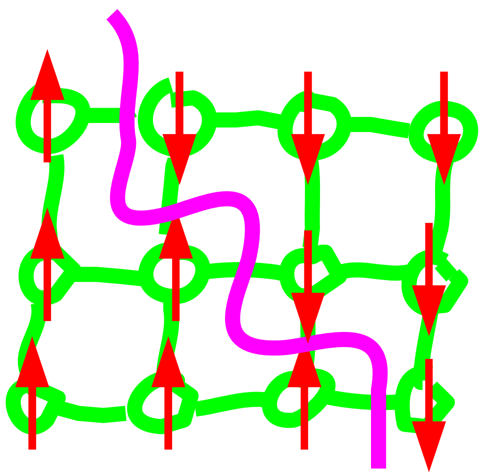
No Phase Transition in 1D

(Peierls)

- Take ordered state, and check if min F, for some T>0

shake ⟺ FLIP some spins
(break order)

<u>2D</u>



$\Delta U = \langle H_{ordered} \rangle - \langle H_{flip} \rangle = 2NJ$

$\Delta S = \log(\text{no. ways to spend}) = \log(3\text{^}N)$

Works for all N

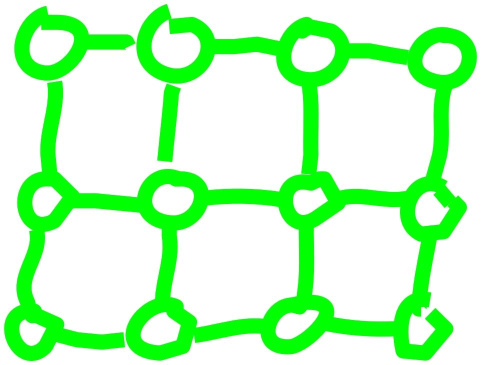
$\Delta F = 2NJ - T\,N\log(3)$

$T_c = 2J / \log(3)$

Measurement Based Quantum Computation (MBQC)

- Resource (multiparty entangled state)

Highly entangled multiqubit state + no knowledge



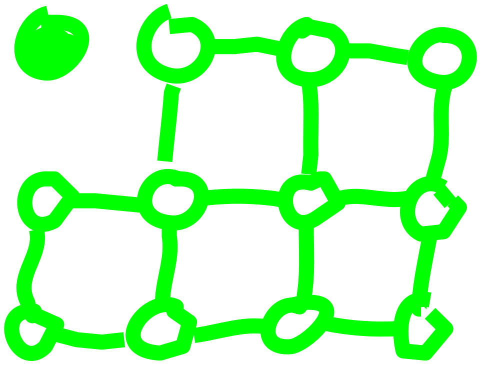
Measurement Based Quantum Computation (MBQC)

- Resource (multiparty entangled state)

Highly entangled multiqubit state + no knowledge

- Process - single qubit measurements
  - local rotations (corrections)

Less entangled multiqubit state + no knowledge

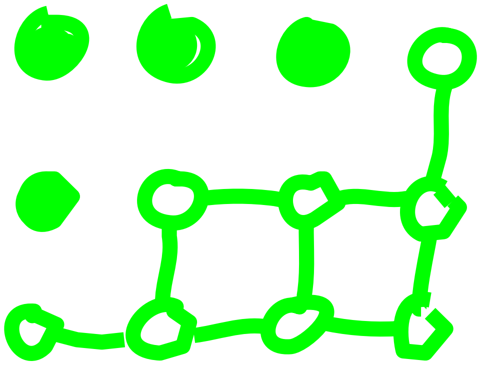


Measurement Based Quantum Computation (MBQC)

- Resource (multiparty entangled state)

Highly entangled multiqubit state + no knowledge

- Process - single qubit measurements
  - local rotations (corrections)



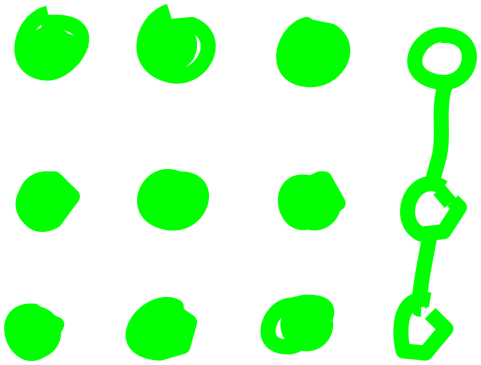
Less entangled multiqubit state + no knowledge

Measurement Based Quantum Computation (MBQC)

- Resource (multiparty entangled state)

Highly entangled multiqubit state + no knowledge

- Process - single qubit measurements
  - local rotations (corrections)

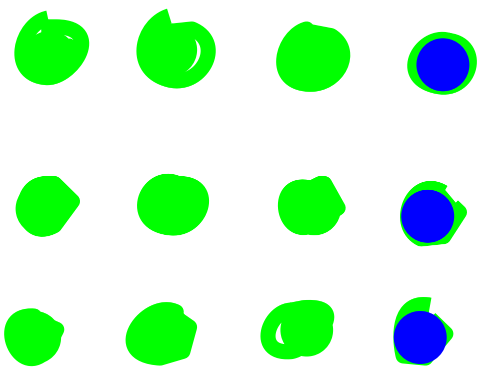


Less entangled multiqubit state + no knowledge

Measurement Based Quantum Computation (MBQC)

- Resource (multiparty entangled state)

Highly entangled multiqubit state + no knowledge



- Process - single qubit measurements
  - local rotations (corrections)

Less entangled multiqubit state + no knowledge

- Read-out - single qubit measurements

Separable multiqubit state + full knowledge = '*solution state*'

Computational 2nd Law. . .

keep computation as 'universal' as possible
(at each time step)

Computational 2nd Law. . .

keep computation as 'universal' as possible
(at each time step)

- Balance:

  energy → **Entanglement** (**E**)

  entropy → '**Computational capacity**' (**C**)
  log (# ways to use entanglement)
  # computations with ent.

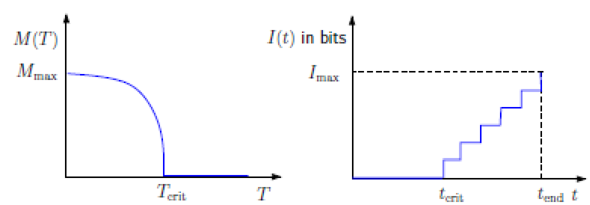
- Temp?

  T → **1/t** inverse number of steps

- 0rdered?

  state magnetised → **state of 'solution'**

  T<Tc M>0 t<tc solution presented



- Free energy?

  Potential $P = E - 1/t\ C$

Computational 2nd Law. . .

keep computation as 'universal' as possible
(at each time step)

- Minimise computational potential

$$P = E - 1/t\, C$$

Entanglement

Capacity = log (number of ways of using E)

Keep entanglement 'most useful'
(at each time step)

Computational 2nd Law. . .

keep computation as 'universal' as possible
(at each time step)

- Minimise computational potential

$$P = E - 1/t \, C$$

Entanglement

Capacity = log(number of ways of using E)

Keep entanglement 'most useful'
(at each time step)

Can I do a universal computation in finite time t? ⇔ Does the 'solution state' have minimum potential (at time t)?

Computational 2nd Law. . .

keep computation as 'universal' as possible
(at each time step)

- Minimise computational potential

P = E – 1/t C

Entanglement (E)

Capacity (C) = log(number of ways of using E)

Keep entanglement 'most useful'
(at each time step)

Can I do a universal computation in finite time t? ⟺ Does the 'solution state' have minimum potential (at time t)?

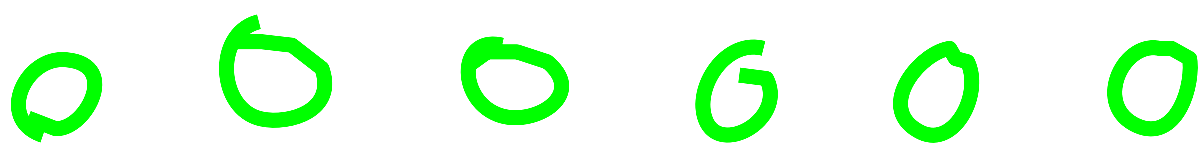
Shake it and see!

Universal resource?

- Take ordered 'solution' state, and check if min P, for some t<∞

shake ⟺ CREATE some entanglement

<u>1D</u>

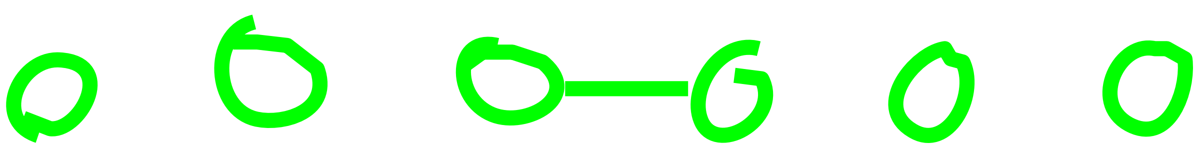


Universal resource?

- Take ordered 'solution' state, and check if min P, for some t<∞

shake ⇔ CREATE some entanglement

<u>1D</u>

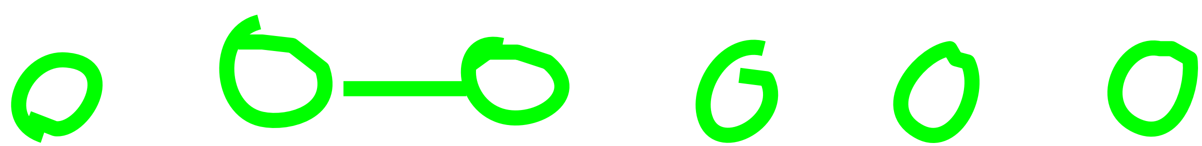


Universal resource?

- Take ordered 'solution' state, and check if min P, for some t<∞

shake ⟺ CREATE some entanglement

1D

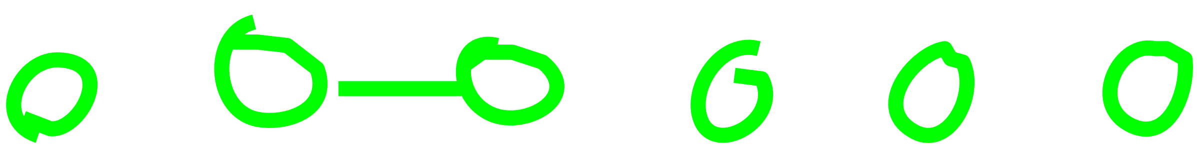


Universal resource?

- Take ordered 'solution' state, and check if min P, for some t<∞

shake ⟺ CREATE some entanglement

<u>1D</u>



ΔE = 1
ΔC = log(no. ways to spend some ΔE) = log(N)

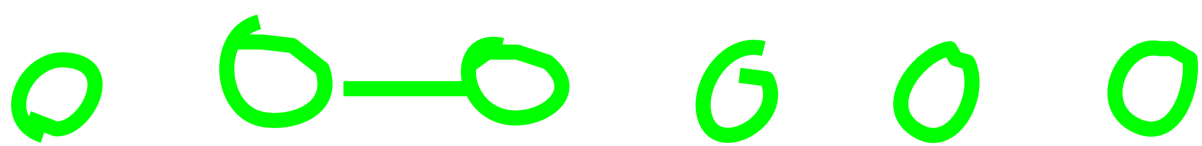
$\Delta P = 1 - 1/t \log(N)$

Universal resource?

- Take ordered 'solution' state, and check if min P, for some t<∞

shake ⟺ CREATE some entanglement

<u>1D</u>



Not universal in finite time!

$\Delta E = 1$

$\Delta C = \log(\text{no. ways to spend some } \Delta E) = \log(N)$

$$\Delta P = 1 - 1/t \log(N)$$

t -> ∞ as N -> ∞

Intuition...

- Our computational 'law of nature' insists to use as universally as possible

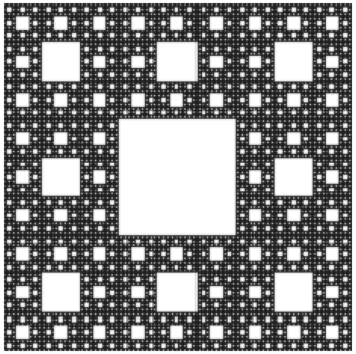
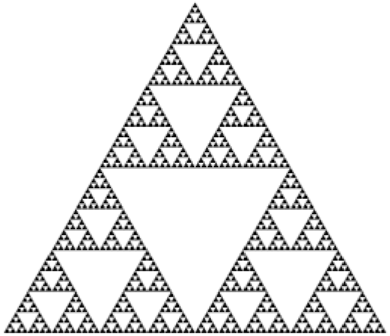
  → keeping optimise way to use each unit of entanglement

- In 1D, using one unit can be done in N ways

$$\Delta P = \Delta E - 1/t\, \Delta C = 1 - 1/t \log(N)$$

  → Loose too many choices using one unit of entanglement: never leads to solution

Fractal Lattices?

	Sierpinski carpet	Sierpinski gasket
		
Dimension	d=1..2	d=1.585
Ramification	R= ∞	R=2 or R=3
		

- Not just dimension that is important, other topological features, ramification, lacunarity, connectivity....

Y. Gefen, B. B. Mandlebrot, and A. Aharony. Phys. Rev. Lett. 45, 855 (1980)

Proofs?

- Proof *of* universality

  - constructive

- Proof *against* universality...?

  - entanglement conditions

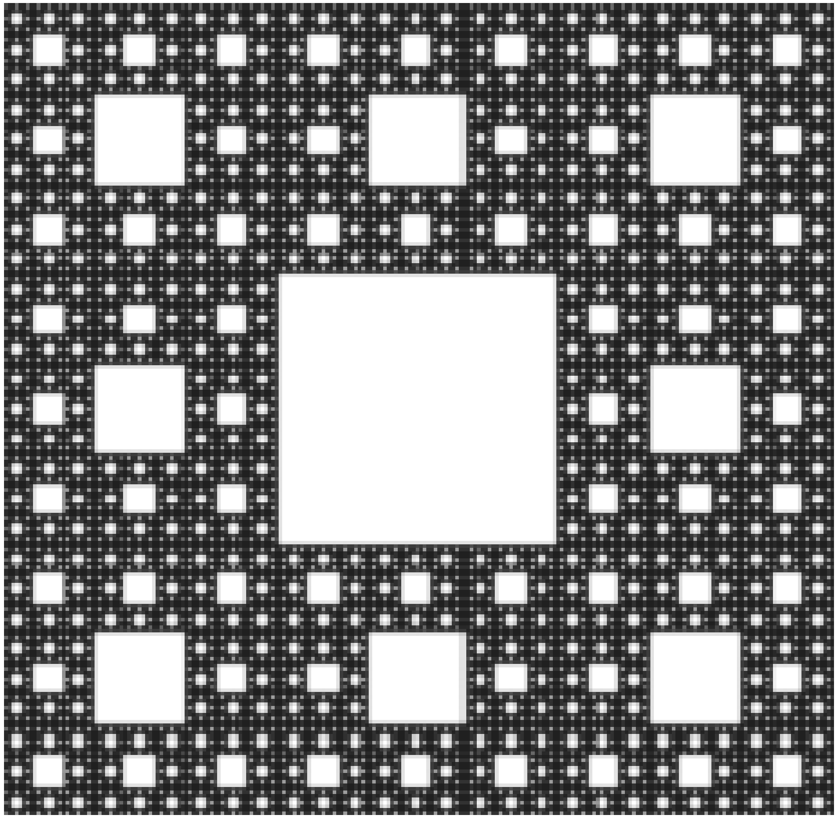
    (proof against universality as state preparator

    of classical simulatibility)

Proof of universality

- Find any 2D grid, measure out the rest*

-> cluster state

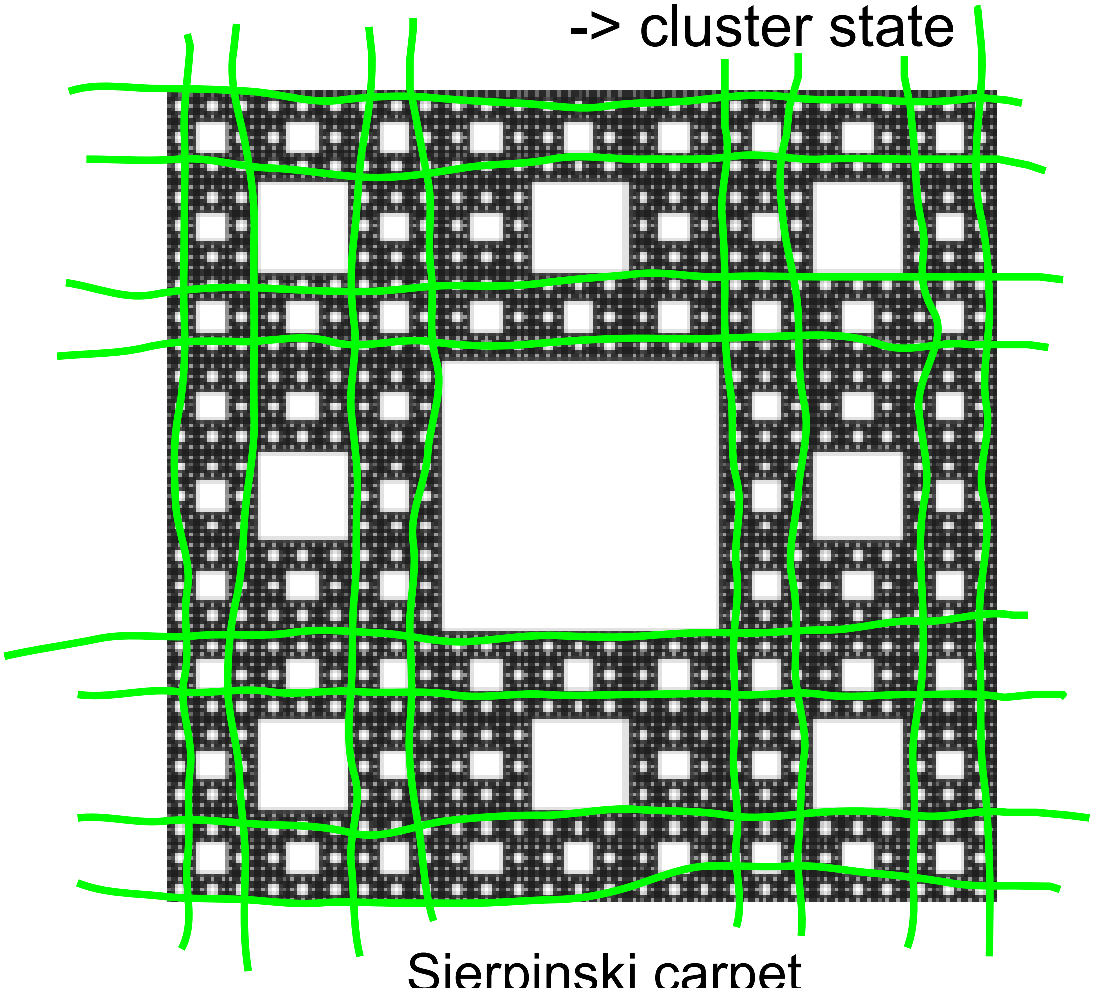


Sierpinski carpet

*Browne et al. New J. Phys. 10, 023010 (2008)

Proof of universality

- Find any 2D grid, measure out the rest*



*Browne et al. New J. Phys. 10, 023010 (2008)

Proof of classical simulatibility

- Entanglement width condition

if $E_{wd}(|\psi_n\rangle) \leq \log(n)$

$\rightarrow$ classically efficiently simulatible / not universal state preparator

* van den Nest et al PRL 97, 150504, (2006).

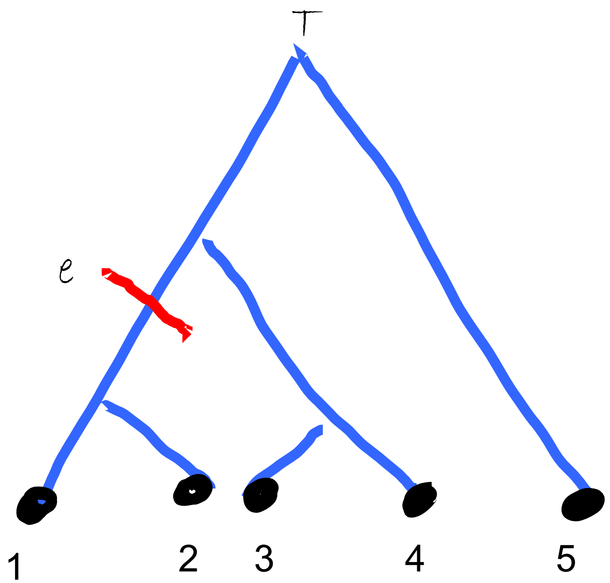
Proof of classical simulatibility

- Entanglement width

$$E_{wd}(|\psi\rangle) := \min_{T} \max_{e} E^{bi}_{T,e}(|\psi\rangle)$$

Subqubic tree T

Cut e



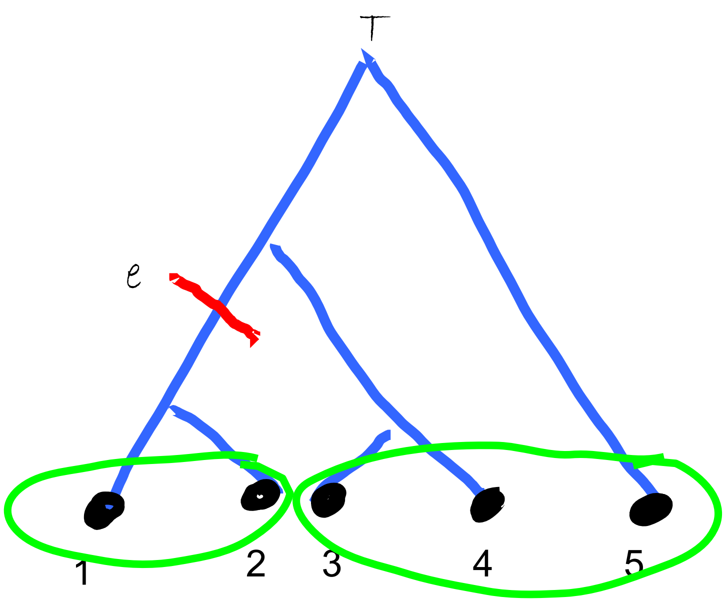
Proof of classical simulatibility

- Entanglement width

$$E_{wd}(|\psi\rangle) := \min_{T} \max_{e} E^{bi}_{T,e}(|\psi\rangle)$$

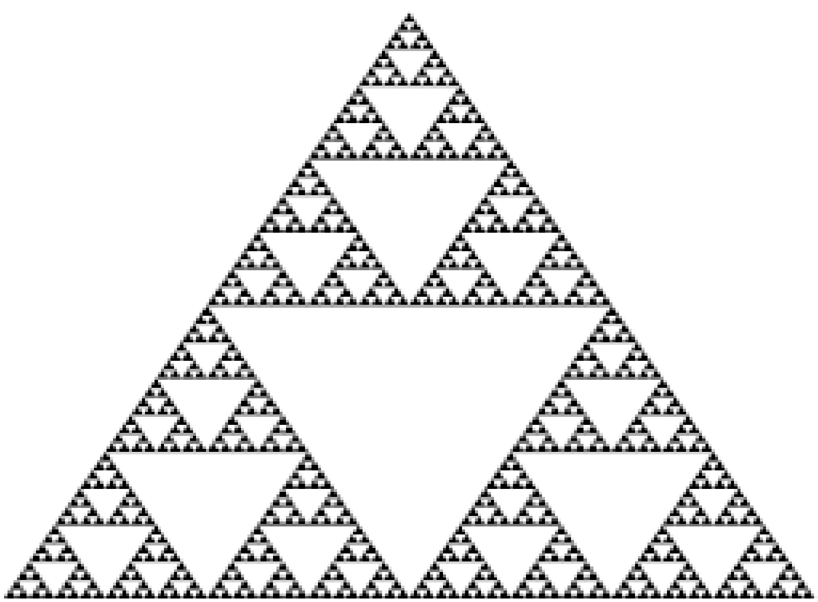
Subqubic tree T

Cut e



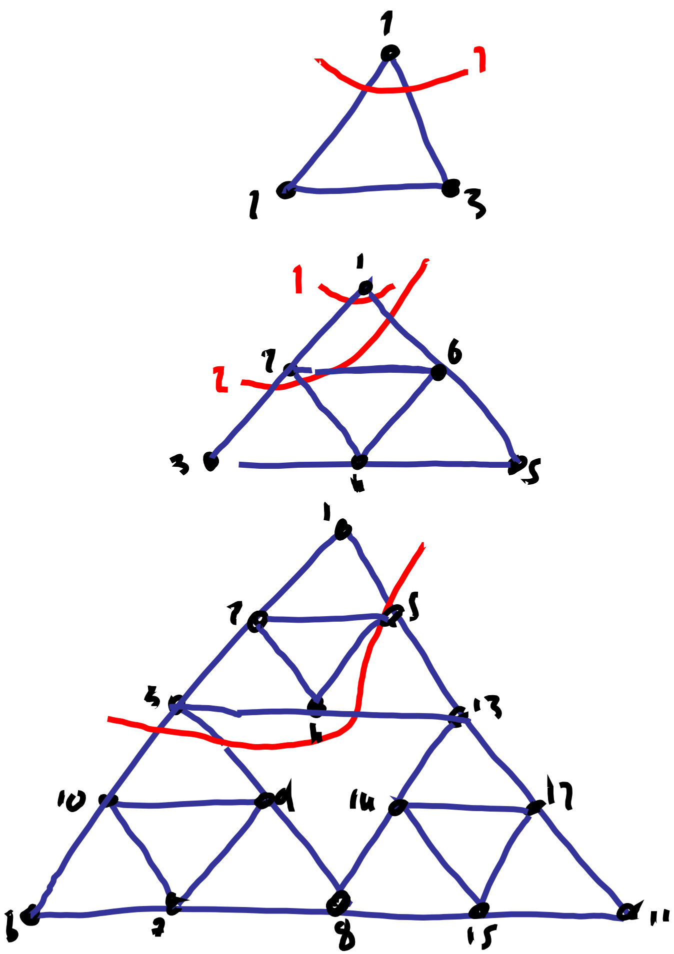
Proof of classical simulatibility

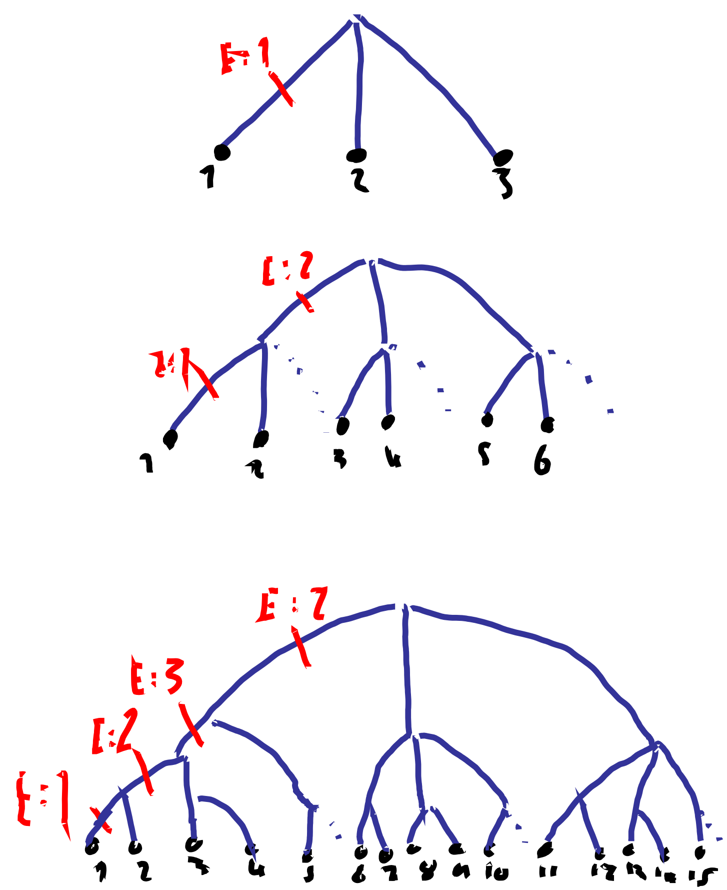
- Entanglement width of Sierpinksi carpet
  - Find self similar trees with same properties (such that entanglement is bounded)



Proof of classical simulatibility

- Entanglement width of Sierpinksi carpet
  - *Find self similar trees with same properties (such that entanglement is bounded)*





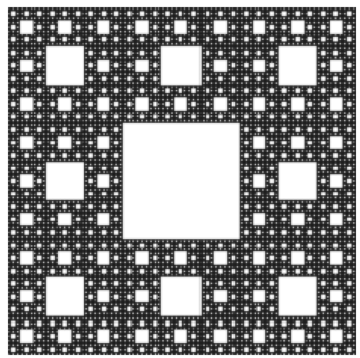
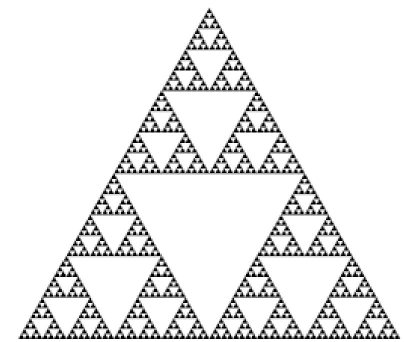
Proof of classical simulatibility

- Entanglement width of Sierpinksi carpet
  - Find self similar trees with same properties (such that entanglement is bounded)

$$E_{wd}(|\psi_{gasket}\rangle) \leq 3$$

  - NOT universal state preparator
  - Classically simulatible

Fractal Lattices

	Sierpinski carpet	Sierpinski gasket
		
Dimension	d=1...2	d=1.585
Ramification	R= ∞	R=2 or R=3
		

- Not just dimension that is important, other topological features, ramification, lacunarity, connectivity....

Conclusions

- Analogy gives insights into features of universal resource
  - Not just dimension, but other topological features (ramification, connectivity...)

- Connections to universality no-gos?
  - Structural entanglement

$$E_{struc}(|\psi_n\rangle) \leq \log(n) \Rightarrow$$ Classically simulatable

    ramification as structural entanglement?
    topological features and classical simulateability?

  - Flow, gFlow

- Extention /solidification of analogy
  - Application to other models of Quantum Computation
  - Path integral approach for QC?